

Solution

PRELIMS 1

Class 10 - Mathematics

Section A

1. (c) rational number  
**Explanation:**  
Given,  $p^2 = \frac{32}{50}$   
 $\Rightarrow p^2 = \frac{16}{25}$   
 $\Rightarrow p = \frac{4}{5}$ , which is a rational number.
2. (a)  $a > 0$ ,  $b < 0$  and  $c > 0$   
**Explanation:**  
Clearly,  $f(x) = ax^2 + bx + c$  represent a parabola opening upwards.  
Therefore,  $a > 0$   
The vertex of the parabola is in the fourth quadrant, therefore  $b < 0$   
 $y = ax^2 + bx + c$  cuts Y axis at P which lies on OY.  
Putting  $x = 0$  in  $y = ax^2 + bx + c$ , we get  $y = c$ .  
So the coordinates of P is (0, c).  
Clearly, P lies on OY.  $\Rightarrow c > 0$   
Hence,  $a > 0$ ,  $b < 0$  and  $c > 0$
3. (a) a unique solution  
**Explanation:**  
**Given:**  $2x + y - 5 = 0$  and  $3x + 2y - 8 = 0$   
We know that the general form for a pair of linear equations in 2 variables x and y is  $a_1x + b_1y + c_1 = 0$  and  $a_2x + b_2y + c_2 = 0$ .  
Comparing with above equations,  
we have  $a_1 = 2$ ,  $b_1 = 1$ ,  $c_1 = -5$ ;  $a_2 = 3$ ,  $b_2 = 2$ ,  $c_2 = -8$   
 $\frac{a_1}{a_2} = \frac{2}{3}$   
 $\frac{b_1}{b_2} = \frac{1}{2}$   
 $\frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$   
Since  $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$   
The lines are intersecting.  
 $\therefore$  The pair of equations has a unique solution.
4. (c)  $ad \neq bc$   
**Explanation:**  
 $(a^2 + b^2)x^2 + 2(ac + bd)x + c^2 + d^2 = 0$   
Here  $A = a^2 + b^2$ ,  $B = 2(ac + bd)$ ,  $C = c^2 + d^2$   
 $D = B^2 - 4AC = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$   
 $= 4[a^2c^2 + b^2d^2 + 2abcd] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$   
 $= 4a^2c^2 + 4b^2d^2 + 8abcd - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2d^2$   
 $= -4a^2d^2 - 4b^2c^2 + 8abcd$   
 $= -4(a^2d^2 + b^2c^2 - 2abcd)$   
 $= -4(ad - bc)^2$   
 $\therefore$  Roots are not real  
 $\therefore D < 0$

$$\therefore -4(ad - bc)^2 < 0 \Rightarrow (ad - bc)^2 < 0$$

$$\Rightarrow ad - bc < 0 \text{ or } ad \neq bc$$

5. (a) -16

**Explanation:**

$n^{\text{th}}$  term from end of an A.P. is,

$a_n = l - (n - 1)d$ , where  $l$ ,  $d$  represents last term and common difference respectively.

Here,  $l = -31$ ,  $d = 2 - 5 = -3$

$$\therefore a_6 \text{ from end} = -31 - (6 - 1)(-3) = -31 + (5 \times 3) = -16$$

6.

(b) 5

**Explanation:**

$$\text{Distance from origin} = \sqrt{(5 - 0)^2 - (0 - 0)^2}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

7.

(d) 3 : 1

**Explanation:**

Let the Pt  $(0, y)$  intersect line segment joining  $(-6, 2)$  &  $(2, -6)$  in ratio  $m : n$  using section formula,

$$(0, y) = \left[ \frac{2m - 6n}{m + n}, \frac{-6m + 2n}{m + n} \right]$$

on comparing obsc.

$$\frac{2m - 6n}{m + n} = 0$$

$$2m - 6n = 0$$

$$2m = 6n$$

$$\frac{m}{n} = \frac{6}{2}$$

$$m : n = 3 : 1$$

8.

(c) DE is not parallel to BC ( $DE \not\parallel BC$ )

**Explanation:**

$$\frac{AD}{BD} = \frac{1.6}{1.8} = \frac{1}{3}$$

$$\frac{AE}{EC} = \frac{1.1}{2.2} = \frac{1}{2}$$

$$\frac{AD}{BD} \neq \frac{AE}{EC}$$

It means converse of BPT is not satisfied.

$\therefore$  DE is not parallel to BC ( $DE \not\parallel BC$ )

9.

(b) 4 cm

**Explanation:**

Join OR & OQ

Now PQOR becomes a quadrilateral

$$\angle QPR = 90^\circ \text{ (given)}$$

$$PQ = PR \text{ (}\because \text{ tangent from external point)}$$

$$OQ = OR = \text{(radius of same circle)}$$

$$\angle OQP = \angle ORP = 90^\circ \text{ (}\because \text{ tangents and radius are perpendicular)}$$

$$\therefore \angle QOR = 360^\circ - \angle OQP - \angle QPR - \angle ORP$$

$$\angle QOR = 360^\circ - 90 - 90 - 90$$

$$\angle QOR = 90^\circ$$

$\therefore$  PQOR becomes a square

sides of a square are same

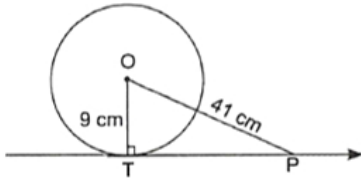
$$\therefore PQ = 4 \text{ cm}$$

10.

**(d)** 40 cm

**Explanation:**

Since tangent at a point on a circle is perpendicular to the radius through the point.



$$\therefore \angle OTP = 90^\circ$$

Now, In right-angled triangle OTP

$$OP^2 = OT^2 + TP^2$$

$$(41)^2 = 9^2 + TP^2$$

$$TP^2 = 1681 - 81$$

$$= 1600$$

$$TP = \sqrt{1600}$$

$$= 40 \text{ cm}$$

11.

**(b)** 1

**Explanation:**

$$\text{Here, } mn = (\sec A + \tan A)(\sec A - \tan A)$$

$$\Rightarrow mn = (\sec^2 A - \tan^2 A) = 1$$

$$[\because \sec^2 \theta - \tan^2 \theta = 1]$$

12.

**(d)**  $30^\circ$

**Explanation:**

$$\text{We have, } 2 \sin 2\theta = \sqrt{3} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$\Rightarrow 2\theta = 60^\circ$$

$$\Rightarrow \theta = 30^\circ$$

13. **(a)**  $30^\circ$

**Explanation:**

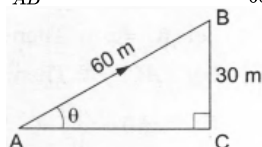
Let AB be the tower and B be the kite.

Let AC be the horizontal and let  $BC \perp AC$ .

Let  $\angle CAB = \theta$ .

BC = 30 m and AB = 60 m. Then,

$$\frac{BC}{AB} = \sin \theta \Rightarrow \sin \theta = \frac{30}{60} = \frac{1}{2} \Rightarrow \sin \theta = \sin 30^\circ \Rightarrow \theta = 30^\circ.$$



14.

(c)  $52 \text{ cm}^2$

**Explanation:**

We know that perimeter of a sector of radius,  $r = 2r + \frac{\theta}{360} \times 2\pi r \dots(1)$

Therefore, substituting the corresponding values of perimeter and radius in equation (1), we get,

$$29 = 2 \times 6.5 + \frac{\theta}{360} \times 2\pi \times 6.5 \dots(2)$$

$$29 = 2 \times 6.5 \left(1 + \frac{\theta}{360} \times \pi\right)$$

$$\frac{29}{2 \times 6.5} = \left(1 + \frac{\theta}{360} \times \pi\right)$$

$$\frac{29}{2 \times 6.5} - 1 = \frac{\theta}{360} \times \pi \dots\dots\dots(3)$$

We know that area of the sector  $= \frac{\theta}{360} \times \pi r^2$

From equation (3), we get

$$\text{Area of the sector} = \left(\frac{29}{2 \times 6.5} - 1\right) r^2$$

Substituting  $r = 6.5$  we get,

$$\text{Area of the sector} = \left(\frac{29}{2 \times 6.5} - 1\right) 6.5^2$$

$$= \left(\frac{29 \times 6.5^2}{2 \times 6.5} - 6.5^2\right)$$

$$= \left(\frac{29 \times 6.5}{2} - 6.5^2\right)$$

$$= \left(\frac{29 \times 6.5}{2} - 6.5^2\right)$$

$$= (94.25 - 42.25)$$

$$= 52$$

Therefore, area of the sector is  $52 \text{ cm}^2$ .

15.

(b)  $\frac{\pi r^2}{4} - \frac{1}{2} r^2$

**Explanation:**

$$\frac{\pi r^2}{4} - \frac{1}{2} r^2$$

16.

(d)  $\frac{1}{3}$

**Explanation:**

p(odd prime no.)

fav case  $\rightarrow 3, 5, 7$

$$p(\text{odd prime no.}) = \frac{3}{9} = \frac{1}{3}$$

17.

(b)  $\frac{3}{4}$

**Explanation:**

At most one Tail

Favourable case  $\rightarrow \text{HH, HT, TH} = 3$

$$p(\text{at most one Tail}) = \frac{3}{4}$$

18.

(d) 20

**Explanation:**

Mean of 2, 7, 6 and  $x = 5$

$$\Rightarrow \frac{2+7+6+x}{4} = 5$$

$$\Rightarrow 15 + x = 20$$

$$\Rightarrow x = 5$$

Also, Mean of 18, 1, 6,  $x$  and  $y = 10$

$$\Rightarrow \frac{18+1+6+x+y}{5} = 10$$

$$\Rightarrow \frac{18+1+6+5+y}{5} = 10$$

$$\Rightarrow 30 + y = 50$$

$$\Rightarrow y = 20$$

19.

**(d)** A is false but R is true.

**Explanation:**

A is false but R is true.

20.

**(c)** A is true but R is false.

**Explanation:**

$$a_{10} = a + 9d$$

$$= 5 + 9(3) = 5 + 27 = 32$$

### Section B

21. By prime factorisation, we get

2	108	2	120	2	252
2	54	2	60	2	126
3	27	2	30	3	63
3	9	3	15	3	21
	3		5		7

$$108 = (2^2 \times 3^3)$$

$$120 = (2^3 \times 3 \times 5)$$

$$252 = (2^2 \times 3^2 \times 7)$$

$$\text{HCF}(108, 120, 252) = \text{product of common terms with lowest power}$$

$$= (2^2 \times 3) = (4 \times 3)$$

$$\text{HCF} = 12$$

$$\text{LCM}(108, 120, 252) = \text{product of prime factors with highest power}$$

$$= (2^3 \times 3^3 \times 5 \times 7)$$

$$\text{LCM} = 7560$$

$$\therefore \text{HCF}(108, 120, 252) = 12$$

$$\text{and } \text{LCM}(108, 120, 252) = 7560.$$

22. It is given that  $AB = 10$  cm,  $AC = 6$  cm and  $BC = 12$  cm

In  $\triangle ABC$ ,  $AD$  is the bisector of  $\angle A$ , meeting side  $BC$  at  $D$

We have to find  $BD$  and  $DC$

Since  $AD$  is  $\angle A$  bisector

$$\text{So } \frac{AC}{AB} = \frac{DC}{BD}$$

Let  $BD = x$  cm

$$\text{Then, } \frac{6}{10} = \frac{12-x}{x}$$

$$\Rightarrow 6x = 120 - 10x$$

$$\Rightarrow 16x = 120$$

$$\Rightarrow x = \frac{120}{16}$$

$$\Rightarrow x = 7.5$$

Now

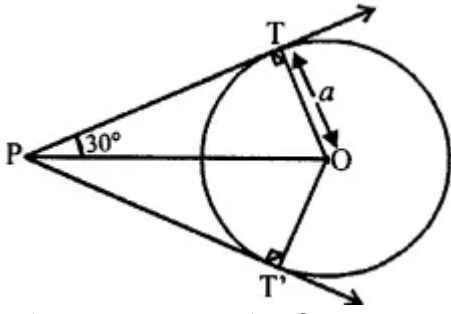
$$DC = 12 - BD$$

$$= 12 - 7.5$$

$$= 4.5$$

Hence,  $BD = 7.5$  cm and  $DC = 4.5$  cm

23. Since, tangents drawn from an external point are equally inclined to the line joining centre to that point.



$$\therefore \angle TPT' = 60^\circ \Rightarrow \angle TPO = 30^\circ$$

Also,  $OT \perp TP$

$$\text{Now, in } \triangle TPO \sin 30^\circ = \frac{OT}{OP}$$

$$\Rightarrow \frac{1}{2} = \frac{a}{OP} \Rightarrow OP = 2a$$

24. We have,  $x = a \sin \theta$  and  $y = b \tan \theta$

$$\therefore \text{LHS} = \frac{a^2}{x^2} - \frac{b^2}{y^2}$$

$$\Rightarrow \text{LHS} = \frac{a^2}{a^2 \sin^2 \theta} - \frac{b^2}{b^2 \tan^2 \theta} \quad [ \because x = a \sin \theta, y = b \tan \theta ]$$

$$\Rightarrow \text{LHS} = \frac{1}{\sin^2 \theta} - \frac{1}{\tan^2 \theta}$$

$$\Rightarrow \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 = \text{RHS} \quad [ \because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \therefore \operatorname{cosec}^2 \theta - \cot^2 \theta = 1 ]$$

OR

$$\cos \theta + \sin \theta = \sqrt{2} \cos \theta$$

$$\Rightarrow \sin \theta = \sqrt{2} \cos \theta - \cos \theta$$

$$\Rightarrow \sin \theta = (\sqrt{2} - 1) \cos \theta$$

$$\Rightarrow \frac{\sin \theta}{\sqrt{2} - 1} = \cos \theta$$

$$\Rightarrow \frac{\sin \theta (\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \cos \theta$$

$$\Rightarrow \sqrt{2} \sin \theta + \sin \theta = \cos \theta$$

$$\Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta = \text{RHS}$$

25. Angle described by the minute hand in 60 minutes =  $360^\circ$

$$\therefore \text{Angle described by the minute hand in 56 minutes} = \left( \frac{360}{60} \times 56 \right)^\circ = 336^\circ$$

$$\therefore \theta = 336^\circ \text{ and } r = 7.5 \text{ cm}$$

$$\therefore \text{Area swept by the minute hand in 56 minutes} = \left( \frac{\pi r^2 \theta}{360} \right)$$

$$= \left( 3.14 \times 7.5 \times 7.5 \times \frac{336}{360} \right) \text{ cm}^2$$

$$= 165 \text{ cm}^2$$

OR

We have,  $r = 16.5 \text{ km}$  and  $\theta = 80^\circ$ .

Let A be the area of the sea over which the ships are warned. Then,

$$A = \frac{\theta}{360} \times \pi r^2 = \frac{80}{360} \times 3.14 \times 16.5 \times 16.5 \text{ km}^2 = 189.97 \text{ km}^2$$

### Section C

26. This problem can be solved using Least Common Multiple because we are trying to figure out when the soonest (Least) time will be that as the event of exercising continues (Multiple), it will occur at the same time (Common).

L.C.M. of 12 and 8 is 24.

So,

They will exercise together again in 24 days.

27. Since  $\alpha, \beta$  are the zeros of the polynomial  $f(x) = x^2 - 5x + k$ .

Compare  $f(x) = x^2 - 5x + k$  with  $ax^2 + bx + c$ .

So,  $a = 1$ ,  $b = -5$  and  $c = k$

$$\alpha + \beta = -\frac{(-5)}{1} = 5$$

$$\alpha\beta = \frac{k}{1} = k$$

Given,  $\alpha - \beta = 1$

$$\text{Now, } (\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$$

$$\Rightarrow (5)^2 = (1)^2 + 4k$$

$$\Rightarrow 25 = 1 + 4k$$

$$\Rightarrow 4k = 24$$

$$\Rightarrow k = 6$$

Hence the value of k is 6.

28. Let the present age of father be x years and sum of present age of two son's be y years.

According to question,

after five years

$$x + 5 = 2(y + 5 + 5)$$

$$x + 5 = 2y + 20$$

$$x - 2y = 15 \dots(i)$$

$$\text{and } x = 3y \dots(ii)$$

$$\therefore 3y - 2y = 15$$

$$\text{or } y = 15$$

$$\therefore \text{age of father } x = 3y$$

$$= 3 \times 15$$

$$= 45 \text{ years}$$

OR

Let the number of oranges with A and B x and y respectively.

Then, according to question,

$$x + 10 = 2(y - 10)$$

$$x + 10 = 2y - 20$$

$$\Rightarrow x - 2y = -30 \dots(1)$$

$$x - y = 20 \dots(2)$$

$$\text{From equation(2), } y = x - 20 \dots(3)$$

substitute this value of y in equation (1), we get

$$x - 2(x - 20) = -30$$

$$\Rightarrow x - 2x + 40 = -30$$

$$\Rightarrow -x = -30 - 40$$

$$\Rightarrow -x = -70$$

$$\Rightarrow x = 70$$

Subtracting this value of x equation (3), we get

$$y = 70 - 20 = 50$$

Hence, the number of oranges with A and B are 70 and 50 respectively.

**Verification:**

Substituting  $x=70, y=50$ , we find that both the equation (1) and (2) are satisfied as shown.

$$x - 2y = 70 - 2(50) = 70 - 100 = -30$$

$$x - y = 70 - 50 = 20 \text{ the verifies the solution.}$$

29.  $TP = TQ$  ...(length of tangents drawn from external points)

$\therefore \angle TQP = \angle TPQ$  (angles oppo to equal sides are equal)

$OP \perp TP$  ( $\because$  at point of contact radius and tangent are  $\perp$ r)

$$\angle OPT = 90^\circ$$

$$\angle OPQ + \angle CPQ = 90^\circ$$

$$\angle TPQ = 90 - \angle OPQ$$

Now, In  $\triangle PTQ$

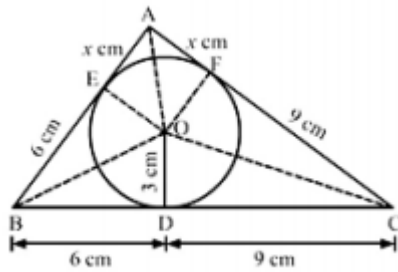
$$\angle TPQ + \angle PTQ + \angle QTP = 180^\circ$$

$$90^\circ - \angle OPQ + 90 - \angle OPQ + \angle PTQ = 180^\circ$$

$$\angle PTQ = 2\angle OPQ$$

Proved.

OR



We know that tangent segments to a circle from the same external point are congruent

Now, we have

$$AE = AF,$$

$$BD = BE = 6 \text{ cm and}$$

$$CD = CF = 9 \text{ cm}$$

Now,

$$\text{Area}(\triangle ABC)$$

$$= \text{Area}(\triangle BOC) + \text{Area}(\triangle AOB) + \text{Area}(\triangle AOC)$$

$$\Rightarrow 54 = \frac{1}{2} \times BC \times OD + \frac{1}{2} \times AB \times OE + \frac{1}{2} \times AC \times OF$$

$$\Rightarrow 108 = 15 \times 3 + (6 + x) \times 3 + (9 + x) \times 3$$

$$\Rightarrow 36 = 15 + 6 + x + 9 + x$$

$$\Rightarrow 36 = 30 + 2x$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3 \text{ cm}$$

$$\therefore AB = 6 + 3 = 9 \text{ cm and}$$

$$AC = 9 + 3 = 12 \text{ cm.}$$

30. Given,

$$\tan A = n \tan B$$

$$\Rightarrow \tan B = \frac{1}{n} \tan A$$

$$\Rightarrow \cot B = \frac{n}{\tan A} \dots\dots\dots(1)$$

Also given,

$$\sin A = m \sin B$$

$$\Rightarrow \sin B = \frac{1}{m} \sin A$$

$$\Rightarrow \operatorname{cosec} B = \frac{m}{\sin A} \dots\dots(2)$$

We know that,  $\operatorname{cosec}^2 B - \cot^2 B = 1$ , hence from (1) & (2) :-

$$\frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$$

$$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow \frac{m^2 - n^2 \cos^2 A}{\sin^2 A} = 1$$

$$\Rightarrow m^2 - n^2 \cos^2 A = \sin^2 A$$

$$\Rightarrow m^2 - n^2 \cos^2 A = 1 - \cos^2 A$$

$$\Rightarrow m^2 - 1 = n^2 \cos^2 A - \cos^2 A$$

$$\Rightarrow m^2 - 1 = (n^2 - 1) \cos^2 A$$

$$\Rightarrow \frac{m^2 - 1}{n^2 - 1} = \cos^2 A$$

31.

Class interval	Mid value (x)	Frequency (f)	fx	Cumulative frequency
0 - 20	10	6	60	6
20 - 40	30	8	240	17
40 - 60	50	10	500	24
60 - 80	70	12	840	36
80 - 100	90	6	540	42



100 - 120	110	5	550	47
120 - 140	130	3	390	50
		N = 50	$\Sigma fx = 3120$	

$$\text{Mean} = \frac{\Sigma fx}{N} = \frac{3120}{50} = 62.4$$

We have,

$$N = 50$$

$$\text{Then, } \frac{N}{2} = \frac{50}{2} = 25$$

The cumulative frequency just greater than  $\frac{N}{2}$  is 36, then the median class is 60 - 80 such that

$$l = 60, h = 80 - 60 = 20, f = 12, F = 24$$

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f} \times h$$

$$= 60 + \frac{25 - 24}{12} \times 20$$

$$= 60 + \frac{20}{12}$$

$$= 60 + 1.67$$

$$= 61.67$$

Here the maximum frequency is 12, then the corresponding class 60 - 80 is the modal class

$$l = 60, h = 80 - 60 = 20, f = 12, f_1 = 10, f_2 = 6$$

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$= 60 + \frac{12 - 10}{2 \times 12 - 10 - 6} \times 20$$

$$= 60 + \frac{40}{8}$$

$$= 65$$

#### Section D

32. Let number of books the shopkeeper buys =  $x$

$$\text{Price of each book} = \text{Rs } \frac{1200}{x}$$

$$\text{cost of each book when } x + 10 \text{ books are bought} = \text{RS } \frac{1200}{x + 10}$$

According to given question,

$$\frac{1200}{x} - \frac{1200}{x + 10} = 20$$

$$1200\left(\frac{1}{x} - \frac{1}{x + 10}\right) = 20$$

$$\left(\frac{1}{x} - \frac{1}{x + 10}\right) = \frac{20}{1200}$$

$$\frac{(x + 10) - x}{x(x + 10)} = \frac{1}{60}$$

$$x + 10 - x = \frac{x^2 + 10x}{60}$$

$$600 = x^2 + 10x$$

$$x^2 + 10x - 600 = 0$$

Here, it is quadratic equation

$$x^2 + 30x - 20x - 600 = 0$$

$$x(x + 30) - 20(x + 30) = 0$$

$$(x + 30)(x - 20) = 0$$

either

$$(x + 30) = 0 \text{ or } (x - 20) = 0$$

$$x = -30 \text{ or } x = 20$$

$x = -30$ , is not possible because the number of books can't be negative.

so, number of books =  $x = 20$ .

OR

Let sides of two squares be  $a$  cm and  $b$  cm

$$\text{Sum of areas of squares} = a^2 + b^2$$

$$\text{Sum of Perimeter} = 4a + 4b$$

$$\text{A.T.Q } a^2 + b^2 = 544$$

$$4a - 4b = 32$$

$$\text{or, } a - b = 8$$

$$a = b + 8$$

$$\Rightarrow a^2 + b^2 = 544$$

$$(b + 8)^2 + b^2 = 544$$

$$\Rightarrow b^2 + 64 + 16b + b^2 = 544$$

$$\Rightarrow 2b^2 + 16b + 64 = 544$$

$$\Rightarrow b^2 + 8b + 32 = 272$$

$$\Rightarrow b^2 + 8b - 240 = 0$$

$$\Rightarrow b^2 + 20b - 12b - 240 = 0$$

$$\Rightarrow b(b + 20) - 12(b + 20) = 0$$

$$b = 12 \text{ or } b = -20$$

Sides cant be -ve

$$b = 12$$

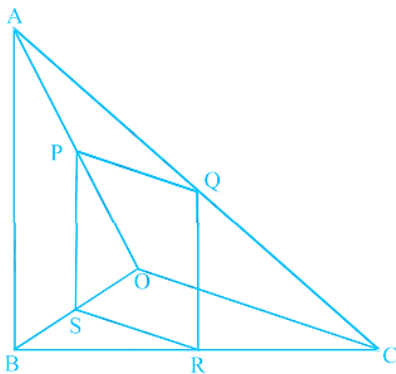
$$a = 20$$

Therefore, sides of two squares are 20 cm and 12 cm respectively

33. It is given that PQRS is a parallelogram,

So,  $PQ \parallel SR$  and  $PS \parallel QR$ .

Also,  $AB \parallel PS$ .



To prove  $OC \parallel SR$

In  $\triangle OPS$  and  $OAB$ ,

$PS \parallel AB$

$\angle POS = \angle AOB$  [common angle]

$\angle OSP = \angle OBA$  [corresponding angles]

$\therefore \triangle OPS \sim \triangle OAB$  [by AAA similarity criteria]

Then,

$$\frac{PS}{AB} = \frac{OS}{OB} \dots(i) \text{ [by basic proportionality theorem]}$$

In  $\triangle CQR$  and  $\triangle CAB$ ,

$QR \parallel PS \parallel AB$

$\angle QCR = \angle ACB$  [common angle]

$\angle CRQ = \angle CBA$  [corresponding angles]

$\therefore \triangle CQR \sim \triangle CAB$

Then, by basic proportionality theorem

$$= \frac{QR}{AB} = \frac{CR}{CB}$$

$$\Rightarrow \frac{PC}{AB} = \frac{CR}{CB} \dots(ii)$$

[ $PS \cong QR$  Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

$$\text{or } \frac{OB}{OS} = \frac{CB}{CR}$$

On subtracting from both sides, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\Rightarrow \frac{OB-OS}{OS} = \frac{CB-CR}{CR}$$

$$\Rightarrow \frac{BS}{OS} = \frac{BR}{CR}$$

By converse of basic proportionality theorem, SR || OC

Hence proved.

34. Radius of hemispherical bowl = radius of cylinder = 7 cm

Height of cylinder = 13 - 7 = 6 cm

Inner surface area of the vessel =  $2\pi rh + 2\pi r^2$

$$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 7(6 + 7)$$

$$= 44 \times 13 = 572 \text{ cm}^2$$

Volume of the vessel =  $\pi r^2 h + \frac{2}{3}\pi r^3$

$$= \pi r^2 (h + \frac{2}{3}r)$$

$$= \frac{22}{7} \times 7 \times 7 (6 + \frac{14}{3})$$

$$= \frac{4928}{3} \text{ cm}^3 \text{ or } 1642.67 \text{ cm}^3$$

OR

CSA of cylinder =  $2 \times \frac{22}{7} \times 2.1 \times 5.8$

$$= 76.56 \text{ cm}^2$$

CSA of two hemisphere =  $4 \times \frac{22}{7} \times 2.1 \times 2.1$

$$= 55.44 \text{ cm}^2$$

Total Surface Area of article =  $76.56 + 55.44 = 132 \text{ cm}^2$

35. p = Frequency of the class + cf of preceding class

$$= 12 + 11 = 23$$

q = cf of the class - cf of preceding class

$$= 46 - 33 = 13$$

Table:

Class Interval	Frequency	Cumulative Frequency
100 - 200	11	11
200 - 300	12	23
300 - 400	10	33
400 - 500	13	46
500 - 600	20	66
600 - 700	14	80

$$N = 80 \Rightarrow \frac{N}{2} = 40$$

The cumulative frequency just greater than 40 is 46.

Hence, median class is 400 - 500.

Here, maximum frequency = 20

Hence, modal class is 500 - 600.

### Section E

36. i. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term,  $a = 30$  and common difference,  $d = 29 - 30 = -1$

Suppose number of rows is  $n$ , then sum of number of bricks in  $n$  rows should be 360.

i.e.  $S_n = 360$

$$\Rightarrow \frac{n}{2} [2 \times 30 + (n - 1)(-1)] = 360 \quad \{S_n = \frac{n}{2}(2a + (n - 1)d)\}$$

$$\Rightarrow 720 = n(60 - n + 1)$$

$$\Rightarrow 720 = 60n - n^2 + n$$

$$\Rightarrow n^2 - 61n + 720 = 0$$

$$\Rightarrow n^2 - 16n - 45n + 720 = 0 \quad [\text{by factorization}]$$

$$\Rightarrow n(n - 16) - 45(n - 16) = 0$$

$$\Rightarrow (n - 16)(n - 45) = 0$$

$$\Rightarrow (n - 16) = 0 \text{ or } (n - 45) = 0$$

$$\Rightarrow n = 16 \text{ or } n = 45$$

Hence, number of rows is either 45 or 16.

$$n = 45 \text{ not possible so } n = 16$$

$$a_{45} = 30 + (45 - 1)(-1) \{a_n = a + (n - 1)d\}$$

$$= 30 - 44 = -14 \text{ [}\because \text{The number of logs cannot be negative]}$$

Hence the number of rows is 16.

ii. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term,  $a = 30$  and common difference,  $d = 29 - 30 = -1$

Suppose number of rows is  $n$ , then sum of number of bricks in  $n$  rows should be 360.

Number of bricks on top row are  $n = 16$ ,

$$a_{16} = 30 + (16 - 1)(-1) \{a_n = a + (n - 1)d\}$$

$$= 30 - 15 = 15$$

Hence, and number of bricks in the top row is 15.

iii. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term,  $a = 30$  and common difference,  $d = 29 - 30 = -1$ .

Suppose number of rows is  $n$ , then sum of number of bricks in  $n$  rows should be 360

Number of bricks in 10th row  $a = 30$ ,  $d = -1$ ,  $n = 10$

$$a_n = a + (n - 1)d$$

$$\Rightarrow a_{10} = 30 + 9 \times -1$$

$$\Rightarrow a_{10} = 30 - 9 = 21$$

Therefore, number of bricks in 10th row are 21.

**OR**

Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term,  $a = 30$  and common difference,  $d = 29 - 30 = -1$ .

Suppose number of rows is  $n$ , then sum of number of bricks in  $n$  rows should be 360.

$$a_n = 26, a = 30, d = -1$$

$$a_n = a + (n - 1)d$$

$$\Rightarrow 26 = 30 + (n - 1) \times -1$$

$$\Rightarrow 26 - 30 = -n + 1$$

$$\Rightarrow n = 5$$

Hence 26 bricks are in 5th row.

37. i. The coordinates of point A are (9, 27), therefore its distance from x-axis = 27 units.

ii. Coordinates of B and C are (4, 19) and (14, 19)

$$\therefore \text{Required distance} = \sqrt{(14 - 4)^2 + (19 - 19)^2}$$

$$= \sqrt{10^2} = 10 \text{ units}$$

iii. Coordinates of F and G are (2, 6) and (16, 6) respectively.

$$\therefore \text{Required distance} = \sqrt{(16 - 2)^2 + (6 - 6)^2}$$

$$= \sqrt{14^2} = 14 \text{ units}$$

**OR**

Coordinates of L and N are (6, 4) and (7, 1) respectively.

$$\text{Length of LN} = \sqrt{(7 - 6)^2 + (1 - 4)^2}$$

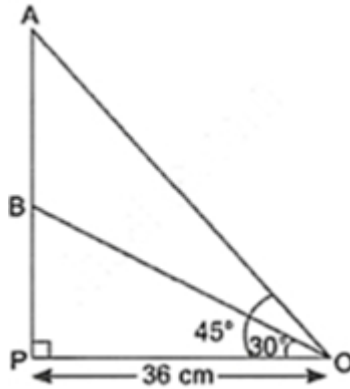
$$= \sqrt{1 + 9} = \sqrt{10} \text{ units}$$

$$\Rightarrow \text{Length of MP} = \sqrt{10} \text{ units}$$

Now, perimeter of LMPN = LN + LM + MP + NP

$$= \sqrt{10} + 6 + \sqrt{10} + 4 = (2\sqrt{10} + 10) \text{ units [}\because \text{LM} = 12 - 6 = 6 \text{ units and NP} = 11 - 7 = 4 \text{ units]}$$

38. i. Let the length of wire BO = x cm



$$\begin{aligned} \therefore \cos 30^\circ &= \frac{PO}{BO} \\ \Rightarrow \frac{\sqrt{3}}{2} &= \frac{36}{x} \\ \Rightarrow x &= \frac{36 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= 12 \times 2\sqrt{3} \\ &= 24\sqrt{3} \text{ cm} \end{aligned}$$

ii. In  $\triangle APO$ ,  $\tan 45^\circ = \frac{AP}{PO}$

$$\begin{aligned} \Rightarrow 1 &= \frac{AP}{36} \\ \Rightarrow AP &= 36 \text{ cm ... (i)} \end{aligned}$$

Now, In  $\triangle PBO$ ,

$$\begin{aligned} \tan 30^\circ &= \frac{BP}{PO} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{BP}{36} \\ \Rightarrow BP &= \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{36}{3} \sqrt{3} \\ &= 12\sqrt{3} \text{ cm} \end{aligned}$$

$$\begin{aligned} \therefore AB &= AP - BP \\ &= 36 - 12\sqrt{3} \text{ cm} \end{aligned}$$

iii. In  $\triangle OPB$

$$\begin{aligned} \tan 30^\circ &= \frac{BP}{PO} \\ \Rightarrow \frac{1}{\sqrt{3}} &= \frac{BP}{36} \\ BP &= \frac{36}{\sqrt{3}} \\ &= \frac{36}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= 12\sqrt{3} \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Now, Area of } \triangle OPB &= \frac{1}{2} \times \text{height} \times \text{base} \\ &= \frac{1}{2} \times BP \times OP \\ &= \frac{1}{2} \times 12\sqrt{3} \times 36 \\ &= 216\sqrt{3} \text{ cm}^2 \end{aligned}$$

**OR**

In  $\triangle APO$ ,  $\tan 45^\circ = \frac{AP}{36}$

$$1 = \frac{AP}{36}$$

$$A = 36 \text{ cm}$$

Height of section A from the base of the tower = AP = 36 cm.